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Title : On the dual topology for certain Lie groups with co-compact nilradical.

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Abstract :

• In the first chapter, we recall the main definitions and properties related to the representation theory of locally compact groups. This includes a focus on nilpotent Lie groups, exponential Lie groups, and compact nilpotent semidirect products. We also discuss the orbit theory established by Lipsman and the topology of the dual space, drawing on J. Dixmier's book on C^* -algebras. Finally, we introduce the concept of the cortex in relation to locally compact groups.

• The second chapter provides a precise description of the Fell topology on the unitary dual \widehat{G}_d of G_d , which, in turn, offers an explicit characterization of the cortex $\text{cor}(G_d)$. These results are detailed in a published article.

In the third chapter, we show that Kirillov-Lipsman orbit mapping

$$\Theta : \widehat{G}_d \rightarrow \mathfrak{g}_d^*/G$$

$$\pi \rightarrow \theta_\pi$$

is a homeomorphism in the setting of the generalized Heisenberg motion groups $G = K \ltimes H_d$.

The results of this chapter are also published in a second article.

• The last chapter is the subject of a submitted article.